E 4259 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2012

Third Semester

Mathematics

Core Course (3)—CALCULUS

(Common for Model I and Model II Mathematics and B.Sc. Computer Applications)

(w.e.f. 2011 Admissions)

(For 2011 Admission Students)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*

Each bunch of **four** *questions carries weight* 1.

I. 1 Find the n^{th} derivative of x^3e^{2x}.

2 Find the Maclaurin's series expansion of $\sinh x$.

3 Write the formula for radius of curvature in Cartesian Co-ordinates.

4 Find the asymptotes parallel to the Co-ordinate axes of $y^2(x^2-a^2)-x=0$.

II. 5 What are points of inflexion ?

6 If $z=y^x$, find $\frac{\partial z}{\partial y}$.

7 If u is a homogeneous function of degree n, then what is the degree of the homogeneous function $\frac{\partial u}{\partial x}$.

8 Write chain rule for partial differentiation.

III. 9 Find the area bounded by the curve $y^2=4ax$, the x–axis and the ordinate $x=h$.

10 Write the Shell formula the volume of the solid generated by the revolution of a region about y-axis.

11 Write the formula for surface area in differential form.

12 State Papus theorem for surface area.

Turn over

IV. 13 State Fubini's theorem, first form.

14 Write the formula for finding the average volume of a function F over a region D in space.

15 What are the Co-ordinate conversion formula for cylindrical to rectangular Co-ordinates ?

16 If $x = r\cos\theta$ and $y = r\sin\theta$, find Jacobian $J(r, \theta)$.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question carries weight 1.

17 If $x = at^2$, $y = 2at$, find $\frac{d^3y}{dx^3}$ in terms of t.

18 Show that $1 - x < e^{-x}$.

19 Find the minimum value of $x^2 + y^2 + z^2$, when $xyz = a^3$.

20 If $u = \frac{xy}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

21 Verify Euler's theorem for $u(x, y) = \sqrt{x} - \sqrt{y}$.

22 Find the area of the region enclosed by $y = x^2 - 2x$ and $y = x$.

23 Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $y = 0$, $x = 2$ about x-axis.

24 Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question carries weight 2.

25 If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_{n+1} - (2n+1)xy_n - n^2y_{n-1} = 0$.

26 Prove that in the curve $y^3 = x(x+2y)$, there is a maximum ordinate at $x = 1$ and that the radius of convergence at this point of $\frac{1}{2}$.

27 Find the shortest and longest distance from the point (1, 2, –1) to the sphere $x^2 + y^2 + z^2 = 24$.

28 Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

29 Use shell method to find the volume of the solid generated by revolving the region bounded by $y = 2x - 1$, $y = \sqrt{x}$, $x = 0$.

30 Evaluate $\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy$.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question carries weight 4.

31 Show that the evolute of

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = \left(a^2 - b^2\right)^{2/3}$.

(b) Find the envelope of the family of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where the two parameters are connected by the relation $a + b = c$.

32 Find the area of the surface generated by revolving the curve $y = x^3 / 9$, $0 \le x \le 2$, x-axis.

33 Evaluate $\int_0^4 \int_{x=y/2}^{x=\frac{y}{2}+1} \frac{2x - y}{2} dx\, dy$.

(2 × 4 = 8)